Properties, Postulates, and Theorems

Postulates: A statement we accept as true without proof "obvious"

Theorems: A statement that is proved by deductive reasoning (uses laws of logic to combine definitions and general statements that we know are true to reach a valid conclusion)

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1. Reflexive Property of Equality- A quantity is equal to itself.

2. Symmetric Property of Equality- An equality may be expressed in either order.

3. Transitive Property of Equality- Quantities equal to the same quantity are equal to each other.

*Congruent segments/angles are segments/angles with equal measures.

Postulates:

1. Substitution- A quantity may be substituted for its equal in any statement of equality.

2. Partition- A whole is equal to the sum of its parts.

3. <u>Addition</u>- If equal quantities are added to equal quantities, the sums are equal.

4. <u>Subtraction</u>- If equal quantities are subtracted from equal quantities, the differences are equal.

5. <u>Multiplication</u>- If equal quantities are multiplied to equal quantities, the products are equal.

*Doubles of equal quantities are equal.

6. <u>Division</u>- If equal quantities are divided to equal quantities, the quotients are equal.

*Halves of equal quantities are equal.

7. <u>Powers</u>- The squares of equal quantities are equal.

8. <u>Roots</u>- Positive square roots of positive equal quantities are equal.

Lesson Practice

Choose the correct answer.

1. Which postulate justifies the following statement?

   If $\angle W \cong \angle X$, then $\angle X \cong \angle W$.

   (1) Transitive Postulate
   (2) Symmetric Postulate
   (3) Substitution Postulate
   (4) Identity Postulate

2. Which postulate would you use in the first step of solving the equation $3a - 19 = 47$?

   (1) Addition Postulate
   (2) Division Postulate
   (3) Substitution Postulate
   (4) Subtraction Postulate

3. The following statement is an example of what postulate?

   If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong EF$.

   (1) Transitive Postulate
   (2) Symmetric Postulate
   (3) Powers Postulate
   (4) Partition Postulate

4. Any quantity is equal to itself. What postulate is this?

   (1) Transitive Postulate
   (2) Symmetric Postulate
   (3) Roots Postulate
   (4) Reflexive Postulate

5. The first two rows of a proof are shown below. What is the missing reason?

Statement	Reason
1. $m\angle 1 + m\angle 2 = 180$ and $180 = m\angle 3 + m\angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	2. ________

   (1) Transitive Postulate
   (2) Symmetric Postulate
   (3) Roots Postulate
   (4) Identity Postulate

6. Which of the following statements about postulates is false?

   (1) The Addition Postulate, Subtraction Postulate, Multiplication Postulate, and Division Postulate are all examples of postulates.
   (2) A postulate has been previously proven true.
   (3) The Roots Postulate states like roots of equal quantities are equal.
   (4) A postulate is accepted as true without proof.

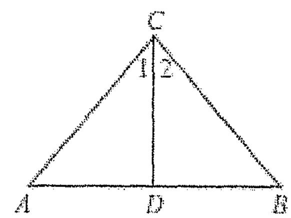
Name ____ Date ____________________

Geometry Pd. _________________ Practice w/ Postulates and Proofs

In 1–6, for each statement, name the postulate that can be used to prove that the conclusion is valid. If more than one postulate is needed, give the correct order. Select from the following list:

a. Transitive property	b. Addition postulate	c. Subtraction postulate
d. Multiplication postulate	e. Division postulate	f. Partition postulate
g. Substitution postulate		

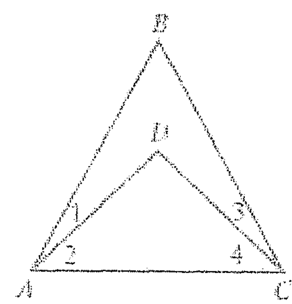
1. If $m\angle 1 + m\angle 2 = 90$ and $m\angle A = m\angle 2$, then $m\angle 1 + m\angle A = 90$. ____________



In 2–3, use the given figure.

2. $m\angle BAC = m\angle 1 + m\angle 2$ ____________

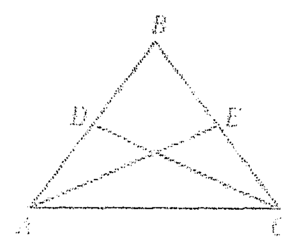
3. If $m\angle BAC = m\angle BCA$ and $m\angle 1 = m\angle 3$, then $m\angle 2 = m\angle 4$. ____________



4. If $AB = CD$, $XY = 2AB$, and $MN = 2CD$, then $XY = MN$. ____________

5. If $AB = \frac{1}{2}CD$ and $GH = \frac{1}{2}CD$, then $\frac{1}{3}AB = \frac{1}{3}GH$. ____________

6. If $m\angle BAC = m\angle BCA$, then $\frac{1}{2}m\angle BAC = \frac{1}{2}m\angle BCA$. ____________



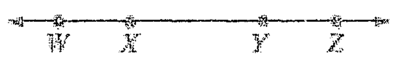
13. *Given:* $9 = 5 + \frac{2}{3}x$

Prove: $x = 6$

Proof:

Statements	Reasons
1. $9 = 5 + \frac{2}{3}x$	1. Given.
2. $9 - 5 = 5 + \frac{2}{3}x - 5$ $4 = \frac{2}{3}x$	2. ______________________
3. $\frac{3}{2}(4) = \frac{3}{2}\left(\frac{2}{3}x\right)$ $6 = x$	3. ______________________
4. $x = 6$	4. ______________________

8. *Given:* $WX = YZ$
Prove: $WY = XZ$



Proof:

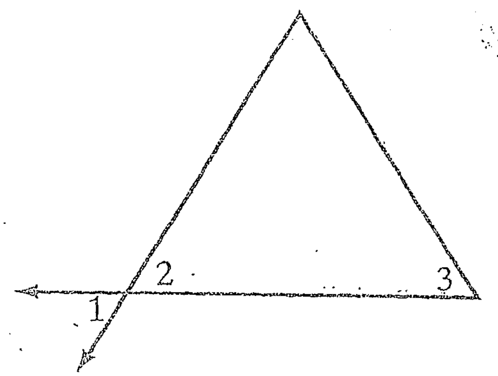
Statements	Reasons
1. $WX = YZ$	1. Given.
2. $XY = XY$	2. ______________________
3. $WX + XY = XY + YZ$	3. ______________________
4. $WX + XY = WY$ $YZ + XY = XZ$	4. ______________________
5. $WY = XZ$	5. ______________________

Postulate Practice

Name the postulate that can be used to prove that each conclusion below is valid.

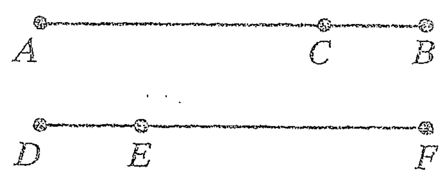
a. Transitive property	b. Addition postulate	c. Subtraction postulate
d. Multiplication postulate	e. Division postulate	f. Partition postulate
g. Substitution postulate		

1. In the given figure, if $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.



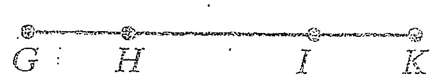
2. If $m\angle A = m\angle B$, then $\frac{1}{2}m\angle A = \frac{1}{2}m\angle B$.

3. In the given figure, if $AB = DF$ and $CB = DE$, then $AC = EF$.

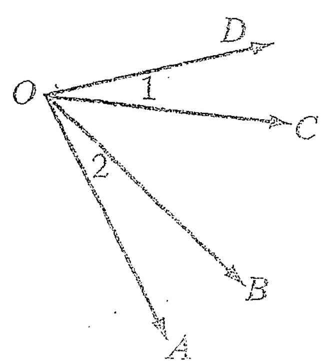


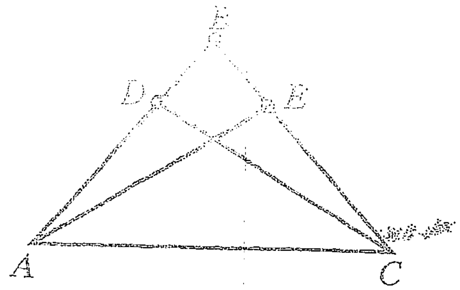
4. If $m\angle H + m\angle J = 180$ and $m\angle J = 40$, then $m\angle H = 140$.

5. In the given figure, if $GH = IK$, then $GI = HK$.



6. In the given figure, if $m\angle 1 = m\angle 2$, then $m\angle AOC = m\angle BOD$.





$\angle B \cong \angle B$ by which postulate?

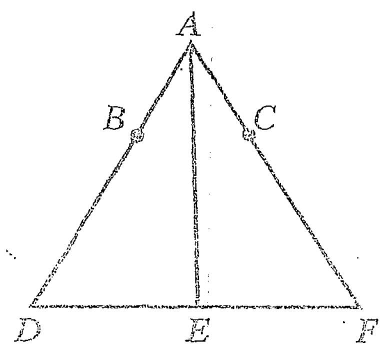
(1) substitution

(2) transitive

(3) reflexive

(4) symmetric

#8-10: $\overline{AD} \cong \overline{AF}$, $\overline{BD} \cong \overline{CF}$, and $\overline{DE} \cong \overline{EF}$ in $\triangle ADF$.



8. Which postulate states that $AB + BD = AD$ and $AC + CF = AF$?

(1) Addition

(2) Partition

(3) Substitution

(4) Subtraction

9. Which postulate states that $AF - CF = AD - BD$ or $AB = AC$?

(1) Addition

(2) Partition

(3) Substitution

(4) Subtraction

10. If $\overrightarrow{AE}$ bisects $\angle DAF$, which of the following is true?

(1) $\angle DEA \cong \angle EAF$

(2) $\angle DAE \cong \angle FAE$

(3) $\overline{DE} \cong \overline{AC}$

(4) $\overline{AB} \cong \overline{DE}$

<u>Properties and Theorems used in Geometry</u>:

- The sum of the measures of the angles in a triangle equals 180 degrees.

- Isosceles Triangle Theorem/Base Angle Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

- Converse of the Isosceles Triangle Theorem: If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

- Vertical angles are congruent.

- The Exterior Angle Theorem: The exterior angle of a triangle is equal to the sum of the two non-adjacent (remote) interior angles.

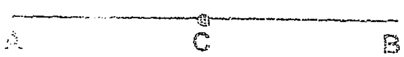
Name: ______________________ Date ______________

Geometry Pd. ________

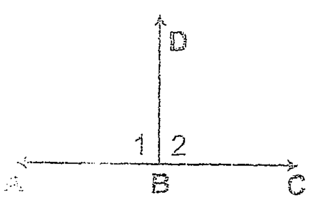
Practice w/Proofs

Questions 1 through 11 refer to the following:

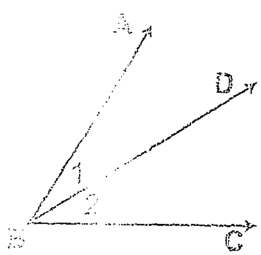
Supply the missing reason(s) for the given proof.



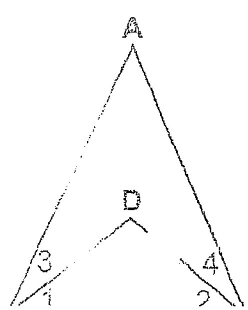
STATEMENTS	REASONS
(1) C is the midpoint of $\overline{AB}$.	(1) Given
(2) $\overline{AC} \cong \overline{CB}$	(2)



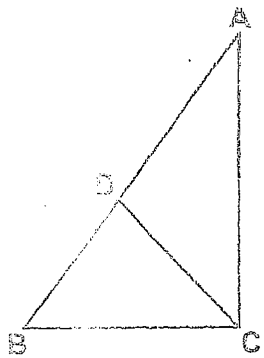
STATEMENTS	REASONS
(1) $\angle 2$ is a right angle.	(1) Given
(2) $\overleftrightarrow{AC} \perp \overrightarrow{BD}$	(2)



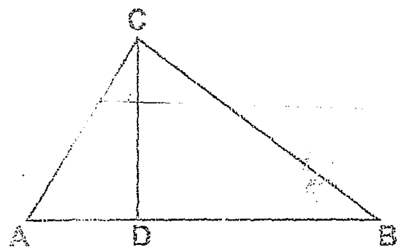
STATEMENTS	REASONS
(1) $\overrightarrow{BD}$ bisects $\angle ABC$	(1) Given
(2) $\angle 1 \cong \angle 2$	(2)



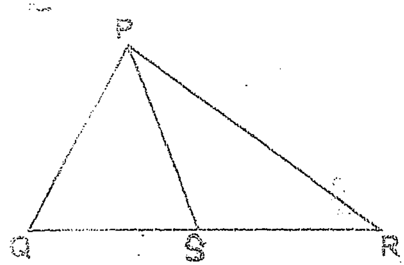
STATEMENTS	REASONS
(1) $m\angle ABC = m\angle ACB$ $m\angle 3 = m\angle 4$	(1) Given
(2) $m\angle 1 \cong m\angle 2$	(2)

5) 

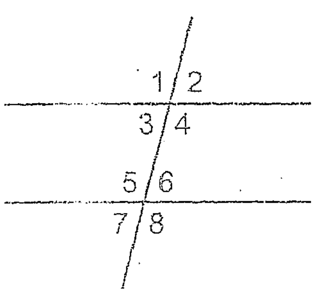
STATEMENTS	REASONS
(1) AD = DC DC = DB	(1) Given
(2) AD = DB	(2)

6) 

STATEMENTS	REASONS
(1) $\overline{CD}$ is the altitude to $\overline{AB}$.	(1) Given
(2) $\overline{CD} \perp \overline{AB}$	(2)

7) 

STATEMENTS	REASONS
(1) $\overline{PS}$ is the median to $\overline{QR}$.	(1) Given
(2) S is the midpoint of $\overline{QR}$	(2)

8) 

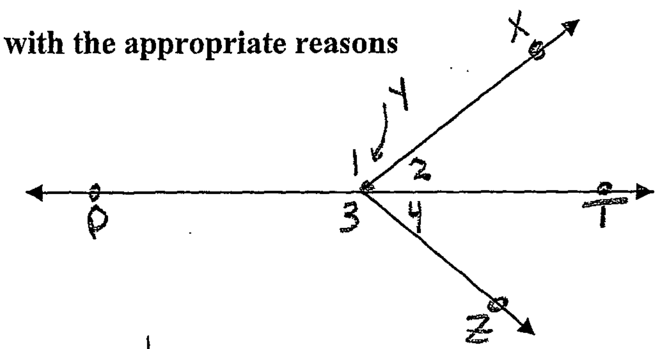
STATEMENTS	REASONS
(1) $m\angle 1 + m\angle 6 = 180°$	(1) Given
(2) $m\angle 6 = m\angle 7$	(2)
(3) $m\angle 1 + m\angle 7 = 180°$	(3)

Name: ________________________________ Date: ____________________

Practicing Proofs

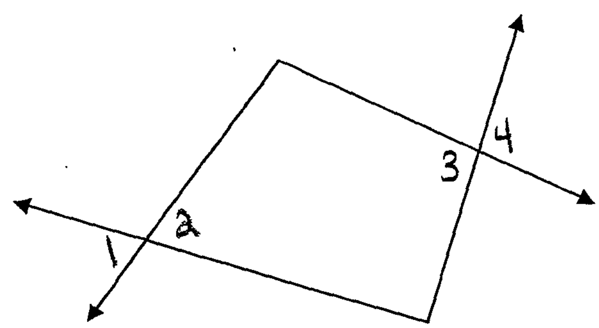
1. **Complete the two-column proof with the appropriate reasons**

Given: $\overline{YT}$ **bisects** $\angle XYZ$
Prove: $m\angle 1 = m\angle 3$

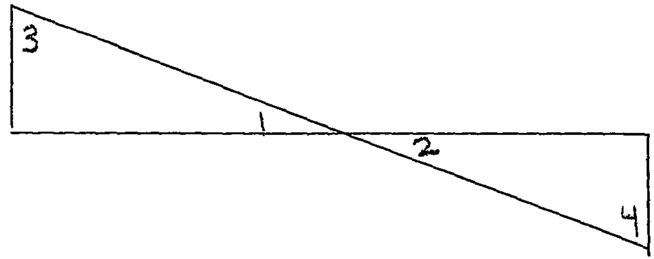


STATEMENTS	REASONS
1.	1.
2. $m\angle 2 = m\angle 4$	2.
3. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 4 = 180$	3.
4. $\angle 1$ and $\angle 2$ are supplementary $\angle 3$ and $\angle 4$ are supplementary	4.
5. $m\angle 1 = m\angle 3$	5.

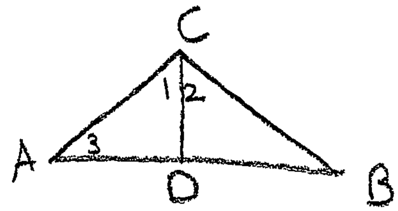
2. **Given:** $\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 3$



3. **Given:** $\angle 3$ **is complementary to** $\angle 1$
$\angle 4$ is complementary to $\angle 2$
Prove: $\angle 3 \cong \angle 4$



4. **Given:** $\overline{AC} \perp \overline{BC}$
$\angle 3$ is complementary to $\angle 1$
Prove: $\angle 3 \cong \angle 2$

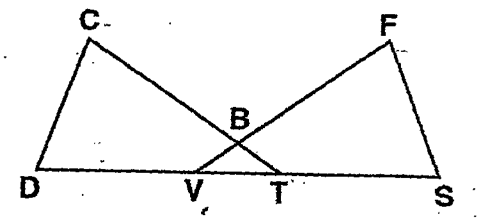


Name ______________________

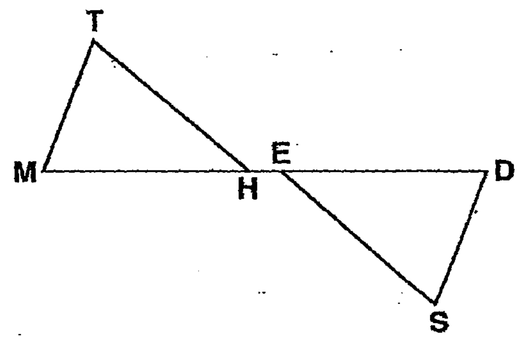
Geometry

Practice with postulates

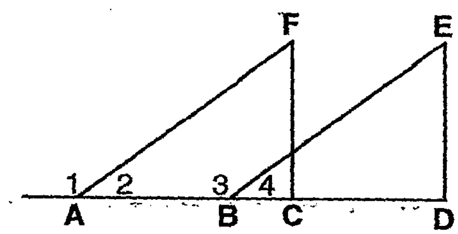
1. Given: $\overline{DV} \cong \overline{TS}$
   Prove: $\overline{DT} \cong \overline{VS}$



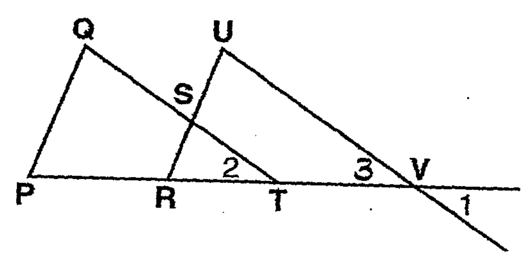
2. Given: $\overline{ME} \cong \overline{HD}$
   Prove: $\overline{MH} \cong \overline{ED}$



3. Given: $\angle 1 \cong \angle 3$
   Prove: $\angle 2 \cong \angle 4$

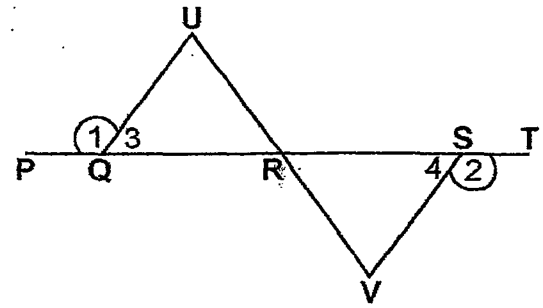


4. Given: $\angle 1 \cong \angle 2$
   Prove: $\angle 2 \cong \angle 3$



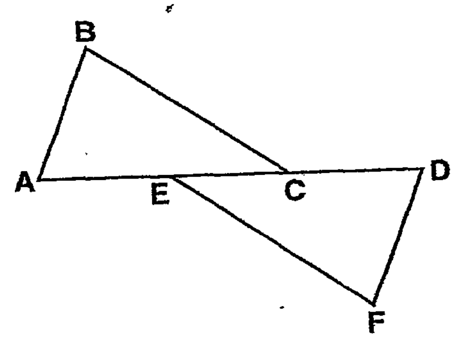
5. Given: <1 ≅ <2

Prove: <4 ≅ <3



6. Given $\overline{AE} \cong \overline{CD}$

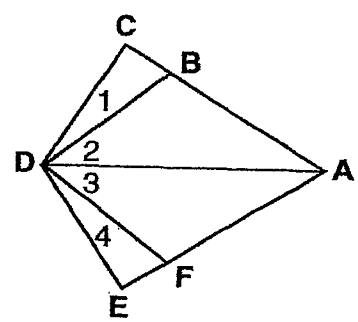
Prove: $\overline{AC} \cong \overline{ED}$



7. Given: <CDA ≅ <EDA

<1 ≅ <4

Prove: <2 ≅ <3



8. Why is <CAD ≅ <BAE?

Why is $\overline{CB} \cong \overline{BC}$?